NUMERICAL OSCILLATIONS IN ELECTROMAGNETIC TRANSIENTS PROGRAMS

Liana CIPCIGAN

Technical University of Cluj-Napoca, Romania

15, C.Daicoviciu str. 3400 Cluj-Napoca, Romania

Liana.Cipcigan@eps.utcluj.ro

Abstract: The Electromagnetic transients progr am and its variants all use the "trapezoidal rule" numerical integration method. The trapezoidal rule is a second order numerical integration technique that is simple to implement. However, it is also susceptible to numerical oscillations when differentiat ing step changes in voltage and current. The program user should be aware of the potential for these oscillations when simulating circuits and systems using EMTP or EMTDC. These oscillations appear, for instance, on the voltage across an inductance after current interruption. The paper presents these oscillations that could occur in EMTP and EMTDC and some options to eliminate them.

Key words: EMTP, EMTDC, numerical oscillations, trapezoidal rule.

1. INTRODUCTION

Transients result in power systems from switching operations, faults, lightning and other disturbances . Simulation programs for electromagnetic transients must provide models for lighting arresters, transformer saturation, circuit breakers and other nonlinear and time-varying effects. Switching and lightning surges, ferroresonance problems and other phenomena of electromagnetic transients have traditionally been studied using transient network analyzers. Digital computer solutions, like EMTP and EMTDC offer an alternative which is very attractive for power systems transient analysis (Glover J.D. and Sarma M., 1994).

The electromagnetic transients program (EMTP) dates back to 1960's development by Hermann Dommel, first in Germany, then at BPA EMTP and has become and industry standard (verified models). EMTP do the following: designed to study transient phenomenon from a few hundred Hertz to hundreds of kHz; switching surges, faults studies, insulation coordination, power electronic interactions with power systems ; EMTP can also model DC systems and electromechanical interactions; outputs are voltage, current, power, and energy versus time; can model simple controls using EMTP's control models or can interface to FORTRAN (in some cases C or Matlab too) (Dommel H.W., 1986).

The electromagnetic transients program dirrect current (EMTDC) has the foloowing features: models for most three phase power apparatus and subsystem (generators, switches, cables, transmission lines, power electronic modules, motors); extensive library of models for controls; interpolation and chatter removal techniques, built-in MATLAB interface controls capabilities; extensive post processing tools.

But, we can say that there is no "world simulator" because most parts of the system have to be simplified.

Analyzing power system transients :
- require understanding of the transient you want to model and of the system;
- require good data to form detailed models;
- develop mathematical model of the system (equivalent RLC circuits);
- solve differential equations–generally a set of cro ss-coupled equations with algebraic constraints.

2. CAUSES OF NUMERICAL OSCILLATIONS

Electromagnetic transients in arbitrary single or multiphase networks are solved by a nodal admittance matrix method. The formulation is based on the method of characte ristics for distributed parameters and the trapezoidal rule of integration for lumped parameters. The digital computer cannot give a continuous history of the transient phenomena, but rather a sequence of snapshot pictures at discrete intervals t. Such discretization causes truncation errors which can lead to numerical instability.

The most common events leading to numerical oscillations are a step change in current through an inductor or a step change in voltage across a capacitor. The cause of this problem can be seen by looking at the differential equations for the inductor and the capacitor.

Inductor voltage $\qquad v = L\dfrac{di}{dt}$ $\qquad$ (1)

Capacitor current $\qquad i = C\dfrac{dv}{dt}$ $\qquad$ (2)

One can think of the numerical oscillations as resulting from forcing an infinite *di/dt* across an inductor or an infinite *dv/dt* on a capacitor.

3. MATHEMATICAL REPRESENTATION

The trapezoidal rule has very good characteristics in terms of low distortion and numerical stability. However, the trapezoidal rule is a-stable, which means that run-off instability cannot occur. For some types of simulations, however, the rule may have to work as a pure differentiator (e.g. voltage on an inductance after current interruption, or current in a capacitance after a voltage is switching on). Under these conditions, sustained (though bounded) numerical oscillations occur (Kahaner D., Moler C., and Nash S., 1989).

The problem can be represented by modeling the equation:

$$x = \frac{dy}{dt} \qquad (3)$$

using the trapezoidal rule. Integrating both sides of the equation result in:

$$y(t) = \int_{t-\Delta t}^{t} x\,dt + y(t - \Delta t) . \qquad (4)$$

The equation rewrite using the trapezoidal rule and with an approximation of the area of the trapezoid bounded by *x*, *t* and *t- t* is:

$$y(t) = y(t - \Delta t) + \frac{\Delta t}{2}(x(t) + x(t - \Delta t)) . \qquad (5)$$

When we solve the equation for *x(t)* this could be the voltage across an inductor:

$$x(t) = -x(t - \Delta t) + \frac{2}{\Delta t}(y(t) - y(t - \Delta t)) . \qquad (6)$$

Start out with *x(t- t)=0* and *y* have undergo a step change from *y(t - t)=0* to *y(t)=1.0*. The function will stay at 1.0 for the rest of the time period of interest. We would expect *y(t)* and *x(t)* to behave as shown in Figure 1.
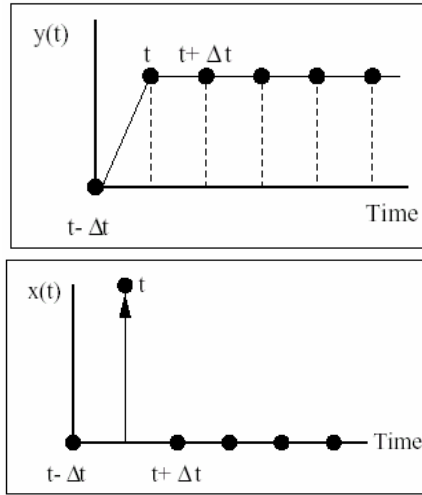

Figure 1 *Plots showing y(t) and x(t).*

Plug the know values into the equation for *x(t)*, resulting in:

$$x(t) = \frac{2}{\Delta t} . \qquad (7)$$

Now step ahead and find *x(t+ t)*:

$$x(t + \Delta t) = -\frac{2}{\Delta t} + \frac{2}{\Delta t}(1.0 - 1.0) = -\frac{2}{\Delta t} . \qquad (8)$$

If we continue ahead another time step, we find that:

$$x(t + 2\Delta t) = \frac{2}{\Delta t} . \qquad (9)$$

This oscillation will continue until circuit conditions change. So, if we plot *x(t)* we see the behavior shown in Figure 2. The average value of *x(t)* is the desired waveform.
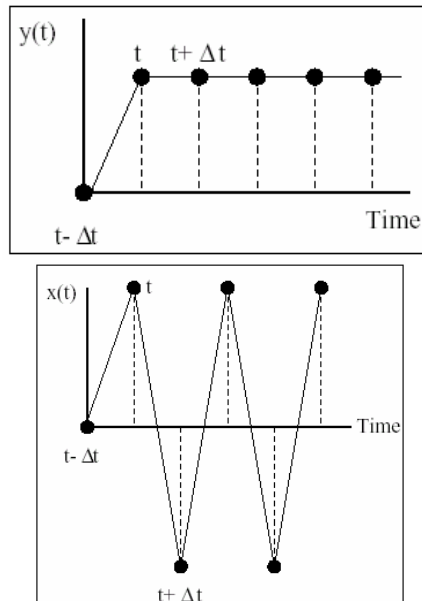

Figure 2 *Actual x(t) with numerical oscillation.*

4. ELIMINATE NUMERICAL OSCILLATIONS

The numerical oscillations can be viewed as arising due to inadequate detail in the system model. For example, oscillations arising from a switch opening in series with an inductor would actually have an arc impedance. However, arc impedance modeling is fairly complex.

Test Case
The test case is an example of a single phase system with a voltage source supplying a resistor through an inductance. A switch connected between the R and the L opens at 1 msec. (Figure 3).
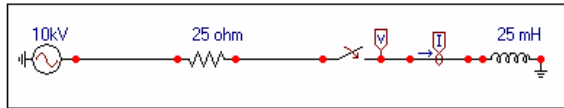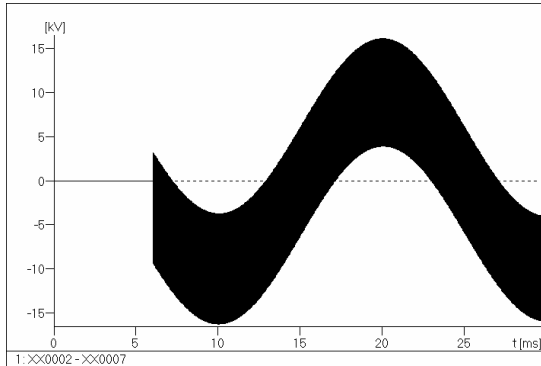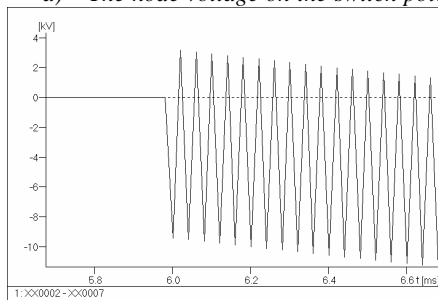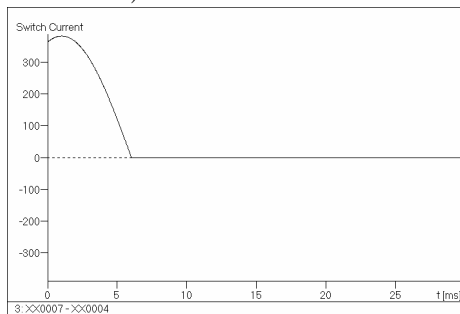
Figure 3 *The EMTP test circuit.*

Figure 4 presents the node voltage on the switch pole (a), the zooms in on the voltage (b) showing the numerical oscillations and the switch current (c).

a) The node voltage on the switch pole.
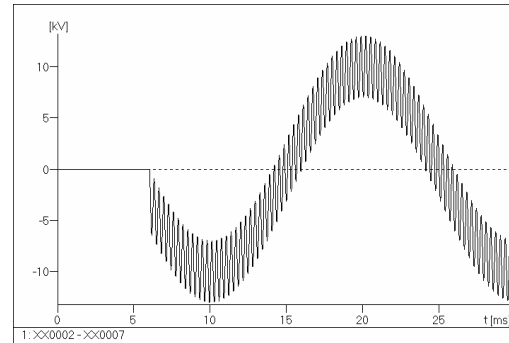
b) Numerical oscillations.

c) Switch current.

Figure 4 *Simulation results.*

Some suggested solutions to eliminate numerical oscillations are to adding circuit elements.

The circuit presented in Figure 3 was first modified to include a parasitic capacitance connected from the inductor side of the switch to ground. As we can see in Figure 5 the oscillations still occur, except this time are now due to the LC resonance and the oscillations are in this case sinusoidal.

a) The node voltage on the switch pole.
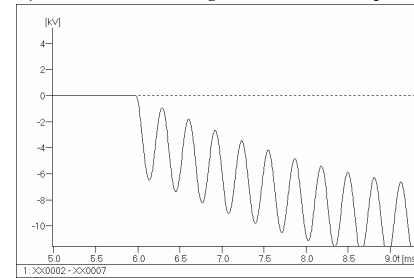
b) Numerical oscillations.

Figure 5 *Simulation results with a capacitance added.*

Figure 6 shows the inductor current, which now has a small sinusoidal oscillation as well. Since the circuit has a very small resistance, these oscillations die slowly.
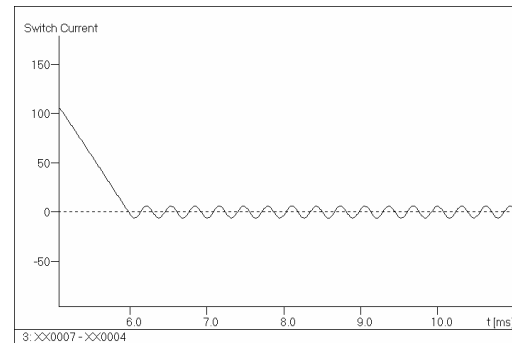
Figure 6 *Inductor current.*

In another cases the program user can add numerical snubbers. These snubbers will also remove or reduce the numerical oscillation problem. Since the snubber circuit provides a bypass path for the switch, the numerical oscillations are avoided, and a more accurate model of the power electronic circuit is used.

These numerical snubbers can also be added for conventional switches as well, although they have less of a basis in reality. If a RC snubber is used the time constant for the RC snubber must still be greater than the simulation time step to avoid problems. The resistance value should be chosen so the RC time constant is a minimum of 2-3 times the simulation time step Δt.

Performance of the RC snubber will vary with the circuit, and the user may need to vary R and C values for best performance. The resulting voltage is presented in Figure 7. Notice that there are no oscillations.
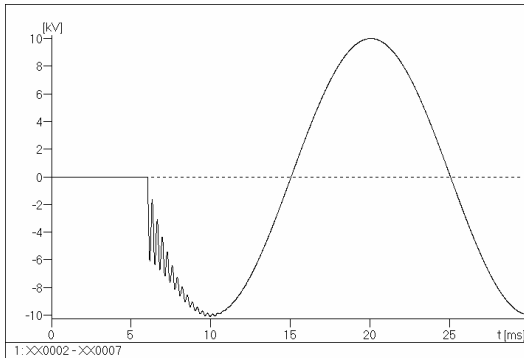

Figure 7 *Resulting voltage with a snubber added.*

A more practical alternative to remove numerical oscillations is the addition of a damping resistor to the circuit. The damping resistor for the inductive switching case should be connected in parallel with the inductor providing a conduction path for the inductor current when the switch opens. The current in the RL circuit will decay with time. Careful choice of the resistance value will lead to rapid damping of the oscillations. However, if the resistance value is too small, the presence of the resistor will impact circuit behavior.
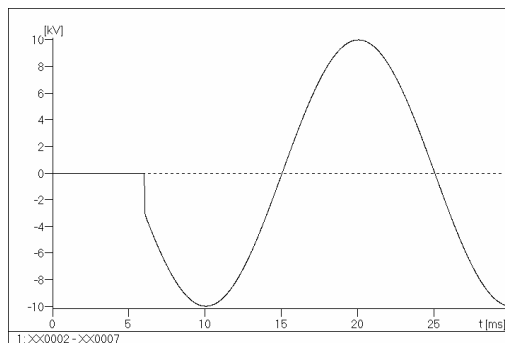

Figure 8 *Resulting voltages with a damping resistor.*

In some cases, reducing the simulation time step can also eliminate numerical oscillations. However, this is not a general purpose solution. It depends on the presence of resistances in the circuit, preferably in parallel with inductances or in series with the capacitances. In many cases the required time step is also far too small for practical simulation.

5. CONCLUSIONS

The techniques presented in this paper coud prevent the numerical oscillations in electromagnetic transients program. The major disadvantage of adding artificial damping either through the integration rule or external resistances, is that the rest of the normal system response is distorted by the phase errors introduced by the damping.

The DCG-EPRI version of EMTP (EMTP version 3.0, also known as EMTP96) eliminates the numerical oscillation problem by performing two time steps using the backward Euler integration method following a discontinuity with time step cut in half. The half-length time steps for the backward Euler stages result in the program restarting with trapezoidal rule integration one timestep after the transition. The user invokes this by typing the keyword "CDA" immediately after the "BEGIN NEW DATACASE" statement. Where "CDA" stands for Critical Damping This method of reducing numerical oscillations is only available in the DCG EPRI version of EMTP (Marti J. And Lin J., 1989).

More complex numerical instabiliy of the EMTP built-in model, the Saturable Transformer Component for the three-winding case, will be discussed in a future work. Through theoretical analysis and computer simulation the source of the numerical instability of the component is identified as the use of the negative inductance in the equivalent circuit of the transformer.

REFERENCES

Dommel H.W., „*Digital computer solution of electromagnetic transients in single and multiphase networks*", IEEE Trans. Power Apparatus and Systems, vol. PAS-88, No.4 pp.388-399, April 1969.
Dommel H.W., „*Electromagnetic Transients Program Reference Manual" (EMTP Theory Book)*, Portland, OR, U.S.A., BPA, August 1986.
Glover J.D. and Sarma M., „*Power System Analysis and Design*", 2nd ed: PWS Publishing Company, 1994.
Kahaner D., Moler C., and Nash S., „*Numerical Methods and Software*", Prentice Hall, Inc., 1989.
Marti J. and Lin J., „*Suppression of numerical oscillations in the EMTP*", IEEE Transactions on Power Systems, Vol.4, No.2, May 1989, pp.739-747

BIOGRAPHY

Liana M. Cipcigan received Ph.D. in Electrical and Computer Engineering from the Technical University of Cluj-Napoca, Romania in 2001. She is working at the Technical University of Cluj-Napoca, Romania, Electrical Power Systems Department from 1995 to present as a Senior Lecturer. Dr. Liana Cipcigan worked as a Post-Doctoral Fellow at the University of Alberta Edmonton, Canada from 2001 to 2002. Her main interests are modeling and simulation of power systems, artificial intelligence techniques applied in power systems, computer aided design and informatic technique.